Electrostatic interaction in dusty plasma

J. Vranješ* and M. Y. Tanaka
National Institute for Fusion Science, 322-6 Oroshi-cho, Toki City 509-5292, Japan

B. P. Pandey
Computational Plasma Dynamics Laboratory, Kettering University, Flint, Michigan 48504

M. Kono
Faculty of Policy Studies, Chuo University, Hachioji, Tokyo 192 0393, Japan
(Received 14 May 2002; published 25 September 2002)

Two modifications of the standard description of electrostatic interaction in a dusty plasma are emphasized. First, the Coulomb-type potential profile is not applicable at very short distances around a dust grain, due to the polarization of the charge on the grain, i.e., the image charge effect, and, second, at larger distances, the standard Debye-Hückel potential screening is modified due to nonlinear corrections in the expanded Boltzmann distribution for plasma particles.

DOI: 10.1103/PhysRevE.66.037401 PACS number(s): 52.27.Lw

In the standard description of electrostatic interaction in dusty plasmas it is assumed that the potential around a dust grain, at distances less than the Debye length r_d, can be approximated by the Coulomb potential, i.e., proportional to $1/r$. However, dust grains behave as isolated conductive (or semiconductive) objects, implying that the charge on grains is generally polarized in the presence of some external field, which can be caused by some approaching charged particle, or by some externally applied constant electric field. We shall emphasize the effects of a charged particle.

From the well known electrodynamics theory [1] it is known that in the simple case of an isolated conducting spherical object with the charge Q, placed at a distance r from a point charge q, the force acting on the point charge q is given by

$$F=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}+F_{ic},\ F_{ic}=-\frac{1}{4\pi\varepsilon_0}\frac{q^2a^3}{r^3}\frac{2r^2-a^2}{(r^2-a^2)^2}. \tag{1}$$

Here a denotes the radius of the spherical object, and the subscript ic denotes the force due to the image charge effect. For $r\gg a$ the image charge term in the above expression is negligible, as it behaves as $\sim 1/r^5$. However, the effect of the image charge becomes dominant for $r\rightarrow a$. This will result in significantly modified (i.e., larger) impact parameters, and, therefore, with larger effective cross sections for the capturing of particles as well. Note that the interaction due to the image charge is always attractive, regardless of the signs of Q and q. It is worth noting that the image charge force exists, and it is always attractive, even for *uncharged*, i.e., neutral dust grains. This fact should be emphasized, especially in applications to astrophysical environments.

A detailed study of cross sections in astrophysical dusty plasma in the presence of image charge effects has been performed [2] as far back as in 1987, but it seems that the study has not caught the attention of the plasma physics community, and, to the best of our knowledge, has not been ever used in any study dealing with the description of collisional processes in dusty plasmas. The numerical calculations for collisional cross sections from Ref. [2] show drastic changes when the image charge force is taken into account, compared to the classical Coulomb-type interaction. The calculations have been done for the conducting and very small grains. It is, however, applicable to the case of dielectric materials as well. For dielectrics there appears a constant of proportionality only [2] of the form $(\epsilon-1)/(\epsilon+2)$, where ϵ is the dielectric constant of the material.

As the collisional cross sections are significantly modified it is easy to see that this effect must be taken into account in any study that involves collisions, such as the charge fluctuations and shadow force effects; for equal sign of charges the standard Coulomb interaction is repulsive, while the image charge effect yields an attractive force acting at short, i.e., collisional distances. The mechanism of the shadow force [3] can be understood from the mechanical point of view. If we have a single dust grain being under bombardment by ions, in principle, there will be no resulting motion of the grain as the bombardment is spherically symmetric. However, when another grain is at some distance r from the first grain, then there will be a solid angle, proportional to a^2/r^2, defining the shadow of the grain by another one, through which the particles will not bombard the grain, resulting in a net momentum transfer, i.e., the "attraction" of the grains. The force of attraction is proportional to the solid angle. As a result we would observe the growth and agglomeration of the grains. This seems to be plausible scenario for the neutrals-grains collisions. However, for the collisions between charged plasma particles and charged grains, or between charged plasma particles and *neutral grains*, due to the

*Author to whom correspondence should be addressed. Permanent address: Institute of Physics, P.O. Box 57, Yu-11001 Belgrade, Yugoslavia. Also at Center for Plasma Astrophysics, Celestijnenlaan 200B, 3001 Leuven, Belgium. Email address: jvranjes@yahoo.com

image charge effects, the forces involved in the process, and consequently, the resulting growth should be completely different. Existing theories predict two types of forces, one repulsive and one attractive, both proportional to $1/r^2$. However, as it follows from Eq. (1), we have a drastic change of r dependence in the presence of the image charge force.

Further, at larger distances in the standard theory it is assumed that the potential is the Debye-Hückel's one, i.e., represented by the function of the form

$$\varphi_{DH}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\exp\left(-\frac{r}{r_d}\right). \tag{2}$$

The potential (2) is obtained by solving the Poisson equation, with the help of the Boltzmann distribution for lighter particles. While it works perfectly for standard plasma, in the case of a dusty plasma this potential profile cannot be correct due to the reasons that will be explained.

In a standard procedure one of the assumptions used is the Boltzmann distribution for particles of the type α (with the charge q_α) in the external potential $\varphi(r)$ (produced by the charge Q),

$$n_\alpha(r)=n_0\exp\left(-\frac{q_\alpha\varphi}{\kappa T_\alpha}\right). \tag{3}$$

Here n_0 is some mean value. Strictly speaking, as is known from statistical theory, the Boltzmann formula (3) is applicable to an ideal gas that is in the state of exact thermodynamical equilibrium, implying that all plasma components have the same temperature.

The second assumption commonly used in the aforesaid solution of the Poisson equation is the assumption that the exponent in Eq. (3) is small everywhere, i.e., the behavior of the plasma particles is close to an ideal gas (except perhaps at a distance very close to the grain, which is then neglected), so that

$$\exp(-y)\approx 1-y,\quad y=\frac{q_\alpha\varphi}{\kappa T_\alpha}, \tag{4}$$

i.e., it is assumed that the electrostatic energy of the particles α is much less than their average thermal energy.

In standard plasmas this is generally valid, but, owing to the large number of electron charge on the grain, for the dusty plasma the expansion (4) should include the higher-order terms, at least to the second order,

$$\exp(-y)\approx 1-y+\frac{1}{2}y^2,\quad y=\frac{q_\alpha\varphi}{\kappa T_\alpha}. \tag{5}$$

Now, applying the standard procedure, we can derive an equation describing potential. We start by the Poisson equation in the form

$$\nabla^2\varphi=-\frac{1}{\varepsilon_0}\sum_\alpha q_\alpha n_\alpha(r), \tag{6}$$

assuming the same temperatures for all plasma components, and using Eq. (5), for the spherical geometry when

$$\nabla^2\varphi\equiv\frac{1}{r}\frac{d^2}{dr^2}(r\varphi).$$

In a few steps we obtain the following equation:

$$\frac{d^2\xi}{dr^2}-\xi+\frac{1}{2r}\xi^2=0. \tag{7}$$

Here $\Sigma_\alpha q_\alpha n_\alpha\approx 0$ is used, as well as the definition of the Debye radius for multicomponent plasma

$$\frac{1}{r_d^2}=\sum_\alpha\frac{1}{r_{d\alpha}^2}=\sum_\alpha\frac{q_\alpha^2 n_\alpha}{\varepsilon_0\kappa T}, \tag{8}$$

and we have introduced the following notation and normalization:

$$\xi\equiv r\frac{q_\alpha\varphi}{\kappa T},\quad r\equiv\frac{r}{r_d},\quad \frac{d^2}{dr^2}\equiv r_d^2\frac{d^2}{dr^2}. \tag{9}$$

For static dust grains the summation in Eq. (8) includes the electrons and ions only. In order to make some numerical estimates we take some typical laboratory values for plasma parameters [4]; let $r_d\approx 5\times 10^{-5}$ m, $T=2\times 10^3$ K, and $Q=eZ$, where $Z=3\times 10^3$. Furthermore, $e=1.6\times 10^{-19}$ C, $\kappa=1.38\times 10^{-23}$ J/K, $\varepsilon_0=8.85\times 10^{-12}$ F/m. Then, at the distance from the grain equal $5r_d$, the Boltzmann exponent in Eq. (5) becomes $y\approx 3.5Z\times 10^{-5}$. Indeed, for ordinary plasma, $Z\sim 1$, the exponent y is small, however, for typical dusty plasma experiments [4], $Z\sim 3\times 10^3$ it becomes $y\approx 0.1$. This confirms that indeed the higher terms in the expansion (5) should be kept in derivations. That issue has been discussed in Ref. [5] for short distances around the grain.

Note that the last term in Eq. (7) is due to the second-order term in the expansion (5). Neglecting it, one immediately obtains the normally used expression for the screened potential in the form

$$\varphi(r)=\frac{c}{r}\exp\left(-\frac{r}{r_d}\right). \tag{10}$$

The constant of integration c is determined from the condition that at small distances, i.e., in the limit $r\rightarrow 0$ the screened potential coincides with the Coulomb potential, yielding therefore Eq. (2). However, in view of what is said above, such an approach is not valid. Namely, in solving Eq. (7) one should bear in mind that, at $r\rightarrow 0$, the solution should coincide with the attractive image charge potential [see Eq. (1)], not with the Coulomb potential. Thus, the complete picture of the potential profile is drastically changed.

The nonlinear Eq. (7) is nontrivial to solve in general. A proper numerical procedure should include boundary conditions of the type $\xi\rightarrow 0$ for $r\rightarrow\infty$ (due to the Debye shielding), and $\xi \to -\infty$ for $r \to 0$ (due to the image charge effect). An approximate solution can be obtained by the following procedure. We may assume a solution that is the sum of the Debye-Hückel-type potential (10), and a perturbation due to the above explained physical reasons, i.e., of the form $\xi = \xi_{DH} + \xi_1$. Putting this into Eq. (7), we obtain the following equation:

$$\frac{d^2\xi_1}{dr^2} - \xi_1 + \frac{1}{2r}(\xi_{DH}^2 + 2\xi_{DH}\xi_1 + \xi_1^2) = 0. \tag{11}$$

Hence, we can neglect the terms comprising ξ_1 in the last term of Eq. (11), and search for an approximate solution of the form

$$\xi_1 = C_1 \exp(r) + C_2 \exp(-r). \tag{12}$$

Now we apply the Lagrange method of variation of parameters, which straightforwardly yields the following set of two equations for the parameters $C_{1,2}$:

$$C_1' \exp(r) + C_2' \exp(-r) = 0, \tag{13}$$

$$C_1' \exp(r) - C_2' \exp(-r) = -\frac{c^2}{2r} \exp(-2r). \tag{14}$$

Here the prime denotes the derivative in respect with r, and c is the constant introduced in Eq. (10). In a few steps one can write the solution for ξ_1 as

$$\xi_1(r) = -\frac{c^2}{4} \mathrm{Ei}(-3r)\exp(r) + \frac{c^2}{4} \mathrm{Ei}(-r)\exp(-r), \tag{15}$$

where $\mathrm{Ei}(-r)$ is the exponential integral [6]

$$\mathrm{Ei}(-r) = -\frac{1}{r}\exp(-r)\left[1 - \frac{1}{r} + \frac{2!}{r^2} - \frac{3!}{r^3} + \cdots + \frac{(-1)^n n!}{r^n}\right] - (-1)^n (n+1)! \int_\infty^r \frac{\exp(-r_1)}{r_1^{n+2}} dr_1 \approx -\frac{1}{r}\exp(-r) \times \left(1 - \frac{1}{r} + \frac{2!}{r^2} - \frac{3!}{r^3}\right).$$

$\mathrm{Ei}(-r)$ is singular for $r \to 0$. However, this is not problematic as we search for solutions at some distances from the grain. Note, at the grain the solution is determined by Eq. (1). Consequently, we can write the new (approximate) solution for the screened potential $\varphi(r)$, normalized in accordance with Eq. (9), as

$$\varphi(r) = \frac{c}{r}\exp(-r) - \frac{c^2}{4}\frac{\exp(r)}{r}\mathrm{Ei}(-3r) + \frac{c^2}{4}\frac{\exp(-r)}{r}\mathrm{Ei}(-r). \tag{16}$$

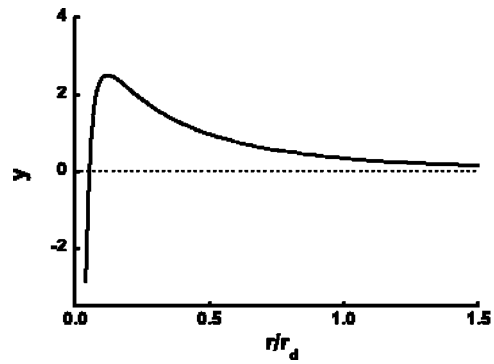

FIG. 1. The profile of the total potential in a dusty plasma, which includes the nonlinear correction to the Boltzmann distribution for large values of r/r_d [Eq. (16)], and the image charge effect at $r \to 0$. Here $y = e\varphi/\kappa T$.

The sketch of the complete potential [i.e., the sum of the potential near the grain, which follows from Eq. (1), and the solution (16)], for the interaction between particles of *the same sign*, is presented in Fig. 1. We see from Fig. 1 that the potential is attractive in $r \to 0$ limit, and changes the sign at some critical r and becomes maximum before decaying.

To conclude, we have explained the physical reasons why in the case of highly charged dust grains the proper description of the potential, at distances that are not too close to the grain, cannot be given by the standard Debye-Hückel potential. Instead, it is more properly described by the nonlinear Eq. (7), which is solved approximately yielding the solution presented by Eq. (16). On the other hand, near the grain the potential is determined by the image charge effect; a fact that has been known from the literature for quite a time, but has not been ever used by researchers. The purpose of the present study is to emphasize these two effects, which should be, consequently, applied in every model describing collisional processes in dusty plasma, i.e., in the properly founded theory of the electrostatic interaction in dusty plasmas. Recently, an improvement of the orbital-motion-limited (OML) theory, which is normally used in the description of dusty plasmas, has been done in Ref. [5], where the trapped ion effect on the shielding has been reexamined. We believe that the effects emphasized in the present study (i.e., the modification of the electrostatic interaction near and far from the grain), which have not been taken into account in Ref. [5], should further improve the OML theory, and give a more detailed description of processes in dusty plasmas.

The importance of these effects is emphasized in the Introduction, but it is seen also from the following. Space and laboratory plasmas can contain grains of the opposite (positive and negative) charges, quite often at the same time, in laboratory dusty plasma [7], cometary dusty plasma [8], noctilucent clouds and the dust in the Earth's middle atmosphere [9], and in space and astrophysical dusty plasmas in general [10]. This peculiar behavior of a dusty plasma is due to several competing processes that are, in principle, present in the system (e.g., attachment of plasma particles due to inelastic collisions, secondary electron emission, dust interaction with photons). At the same time, the amount of charge on different particles can be different. Finally, the image charge force is active even for uncharged conductive or semiconductive grains. Consequently, the importance of the image charge effect and the nonlinear corrections in the Boltzmann distribution is not uniform for every pair of interacting charged bodies (grain plus a plasma particle). The contribution of these effects will depend on the amount of the charge on the given grain, as well as on the signs of charges on the grain and the given plasma particle.

[1] J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), pp. 54–64.
[2] B.T. Draine and B. Sutin, Astrophys. J. **320**, 803 (1987).
[3] R. Bingham and V.N. Tsytovich, Astron. Astrophys. **376**, L43 (2001).
[4] A. Barkan, N. D'Angelo, and R.L. Merlino, Phys. Rev. Lett. **73**, 3093 (1994).
[5] D.D. Tskhakaya, P.K. Shukla, and L. Stenflo, Phys. Plasmas **8**, 5333 (2001).
[6] J. Mathews and R.L. Walker, *Mathematical Methods of Physics* (Benjamin, New York, 1970), pp. 77–82.
[7] N. D'Angelo, Planet. Space Sci. **49**, 1251 (2001).
[8] T.A. Elis and J.S. Neff, Icarus **91**, 280 (1991).
[9] O. Havnes, T. Aslaksen, and A. Brattli, Phys. Scr. **T89**, 133 (2001).
[10] M. Horanyi and C.K. Goertz, Astrophys. J. **361**, 155 (1990); V.W. Chow, D.A. Mendis, and M. Rosenberg, J. Geophys. Res., [Planets] **98**, 19 065 (1993); N. Meyer-Vernet, Astron. Astrophys. **105**, 98 (1982).